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Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics. The Temple College, Philadelphia, Pa.

Let $FG=ED=r$, $FD=2A$, $GE=2a$, $\angle EGK=\varphi$, $\angle DFH=\theta$, $\angle GLF=x$, $\angle CGK=\angle CFH=\delta$, $CN=c$, $CM=d$.

Then $GF:LF=\sin x:\sin LGF$, or $r:LF=\sin x:\sin(\delta-\varphi)$.

Also $ED:LD=\sin x:\sin LED$, or $r:2A+LF=\sin x:\sin(\delta+\varphi)$.

$\therefore LF\sin x=r\sin(\delta-\varphi)=r\sin(\delta+\varphi)-2A\sin x$.

$\therefore A\sin x=r\cos\delta\sin\varphi$.

Similarly, $a\sin x=r\cos\delta\sin\theta$. $\therefore A\sin\theta=a\sin\varphi$.

$GN+EN=2a=2c\cos\delta\sin\delta\cos\varphi/(\cos^2\varphi-\cos^2\delta)$.

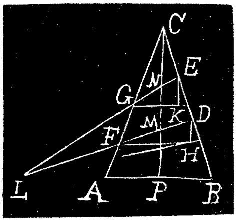
$\therefore c=a(\cos^2\varphi-\cos^2\delta)/\cos\delta\sin\delta\cos\varphi$,

$d=A(\cos^2\theta-\cos^2\delta)/\cos\delta\sin\delta\cos\theta$.

The axes of the projections on any plane perpendicular to CP are $A\cos\theta$ and B, and $a\cos\varphi$ and b, respectively.

$$\left.\begin{array}{l}x^2\cos^2\varphi(\tan^2\delta-\tan^2\varphi)+y^2\tan^2\delta+2cx\sin\varphi=c^2\\ x^2\cos^2\theta(\tan^2\delta-\tan^2\theta)+y^2\tan^2\delta+2dx\sin\theta=d^2\end{array}\right\}\text{are the equations to the ellipses.}$$

$$\therefore A^2=\frac{d^2\cos^2\theta\sin^2\delta\cos^2\delta}{(\cos^2\theta-\cos^2\delta)^2},\ B^2=\frac{d^2\cos^2\theta\cos^2\delta}{\cos^2\theta-\cos^2\delta}.$$



$$a^2=\frac{c^2\cos^2\varphi\sin^2\delta\cos^2\delta}{(\cos^2\varphi-\cos^2\delta)^2}=b^2=\frac{c^2\cos^2\varphi\cos^2\delta}{\cos^2\varphi-\cos^2\delta}.$$

Substituting values of d and c,

$$A^2\cos^2\theta-B^2=\frac{d^2\cos^2\theta\sin^2\theta\cos^4\delta}{(\cos^2\theta-\cos^2\delta)^2}=A^2\sin^2\theta\cot^2\delta.$$

$$a^2\cos^2\varphi-b^2=\frac{c^2\cos^2\varphi\sin^2\varphi\cos^4\delta}{(\cos^2\varphi-\cos^2\delta)^2}=a^2\sin^2\varphi\cot^2\delta=A^2\sin^2\theta\cot^2\delta.$$

$\therefore A^2\cos^2\theta-B^2=a^2\cos^2\varphi-b^2$, and the sections are confocal.

202. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The equations $\sqrt{[l\alpha]}+\sqrt{[m\beta]}+\sqrt{[n\gamma]}=0$ and $l\beta\gamma+m\alpha\gamma+n\beta\alpha=0$ represent ellipses. If a, b, c are the sides of the triangle of reference, transform to Cartesian coördinates and find area of each ellipse.

Solution by the PROPOSER.

$\sqrt{[l\alpha]}+\sqrt{[m\beta]}+\sqrt{[n\gamma]}=0$, or

$$l^2\alpha^2+m^2\beta^2+n^2\gamma^2-2mn\beta\gamma-2nl\alpha\gamma-2ln\alpha\beta=0.....(1),$$

$$l\beta\gamma+m\gamma\alpha+n\alpha\beta=0.....(2).$$

Let ABC be the triangle of reference, B the origin, BC, BA the axes of coördinates. Then $\alpha=y\sin B$, $\gamma=x\sin B$, also $a\alpha+b\beta+c\gamma=ac\sin B$.

$\therefore \beta=[ac\sin B-a\alpha-c\gamma]b=\sin B[ac-ay-cx]/b$.

These values of α, β, γ in (1) and (2) give

$$[bn+cm]^2x^2+[am+bl]^2y^2+2[acm^2+abmn+bclm-b^2nl]xy$$

$$-2[ac^2m^2+abcmn]x-2[a^2cm^2-abclm]y+a^2c^2m^2=0\ldots\ldots(3).$$

$$clx^2+any^2+[al+cn-bm]xy-aclx-acny=0\ldots\ldots(4).$$

(3) reduces to $\{[bn+cm]x+[am+bl]y-acm\}^2-4b^2nlxy=0$.

If $A'x^2+B'y^2+2H'xy+2G'x+2F'y+C'=0$ be the equation (general) to an ellipse and $\triangle$ its discriminant, then area$=\dfrac{\pi\triangle\sin B}{[A'B'-H'^2]^{\frac{3}{2}}}$ where the axes are inclined at an angle B.

For (3), $\triangle=-4a^2b^4c^2l^2m^2n^2$, $A'B'-H'^2=4b^2lm[abn+acm+bcl]$.

$$\therefore \text{Area}=\tfrac{1}{2}\pi a^2bc^2\sin B\sqrt{\frac{lmn}{[abn+acm+bcl]^3}}.$$

If $l=\cos^2\frac{1}{2}A$, $m=\cos^2\frac{1}{2}B$, $n=\cos^2\frac{1}{2}C$, the ellipse becomes the in-circle.

$$\text{Area}=\frac{\pi a^2bc^2\cos\frac{1}{2}A\ \cos\frac{1}{2}B\ \cos\frac{1}{2}C\ \sin B}{2\surd\{[bc\cos^2\frac{1}{2}A+ac\cos^2\frac{1}{2}B+ab\cos^2\frac{1}{2}C]^3\}}$$

$$=\frac{\pi acs\surd\{s[s-a][s-b][s-c]\}\sin B}{2\surd(\{[s-a]+s[s-b]+s[s-c]\}^3)},\ \text{where } s=\tfrac{1}{2}[a+b+c],$$

$$=\tfrac{1}{2}\pi ac\sin B\surd\{s[s-a][s-b][s-c]\}/s^2$$

$$=\pi\left(\frac{\text{area of triangle}}{s}\right)^2=\pi r^2$$

For (4), $\triangle=-\frac{1}{2}a^2bc^2lmn$, $A'B'-H'^2=\frac{1}{4}\{4acln-[al+cn-bm]^2\}$.

$$\text{Area}=\frac{2\pi a^2bc^2lmn\sin B}{\surd([\{4acln-[al+cn-bm]^2\}^3)}.$$

Let $l=\sin A$, $m=\sin B$, $n=\sin C$, and the ellipse becomes the circum-circle.

$$\text{Area}=\frac{2\pi a^2bc^2\sin A\ \sin^2B\ \sin C}{\{4ac\sin A\sin C-[a\sin A+c\sin C-b\sin B]\}^{\frac{3}{2}}}$$

$$=\frac{2\pi a^2b^3c^3\sin A}{\sqrt{(\{4a^2c^2-[a^2+c^2-b^2]^2\}^3)}}=\frac{\pi a^2b^3c^3\sin A}{32\sqrt{(\{s[s-a][s-b][s-c]\}^3)}}$$

$$=\frac{\pi a^2b^2c^2}{[4\text{ area of triangle}]^2}=\pi R^2.$$

CALCULUS.

160. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A dog at the vertex of a right conical hill pursues a fox at the foot of the hill How far will the dog run to catch the fox, if the dog runs directly towards the fox at all times, and the fox is continually running around the hill at its foot, the velocity of the dog being 6 feet per second, the velocity of the fox being 5 feet per second, the hill being 100 feet high and 200 feet in diameter at the base?

Note by J. E. SANDERS, Hackney, Ohio.

The latter part of the solution given in the April MONTHLY is not correct. The dog cannot run towards the fox at all times and keep between the fox and the vertex of the hill. If we suppose only that he runs so that he is directly between the fox and the vertex, the following is the solution:

Let the surface of the hill be rolled out flat. Then the radius of the fox's path$=100\sqrt{2}$ feet$=a$. Put $n=\frac{5}{6}$, $s=$length of dog's path.

$$ds=[a/n]d\theta=\sqrt{\{r^2+[dr/d\theta]^2\}}.$$

$\therefore d\theta=\dfrac{ndr}{\sqrt{[a^2-n^2r^2]}}$, which gives $\theta=\sin^{-1}[nr/a]$ when integrated. When the dog catches the fox $r=a$, and

$\therefore \theta=\sin^{-1}n$ and $s=[a/n]\theta=120\sqrt{2}\sin^{-1}[\frac{5}{6}]=167.178$ feet.

No solution of Problem 165 has been received. See April number, 1903, page 115.

166. Proposed by T. N. HAUN, Mohawk, Tenn.

Find the volume of the solid formed by the revolution of the curve $[y^2+x^2]=a^2[x^2-y^2]$ round the axis of x.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$$[y^2+x^2]^2=a^2[x^2-y^2].$$

$$\therefore y^4+2x^2y^2+x^4=a^2x^2-a^2y^2.\quad \therefore y^2=\tfrac{1}{2}\{a\sqrt{[a^2+8x^2]}-a^2-2x^2\}.$$